

## Lecture 4 - January 19

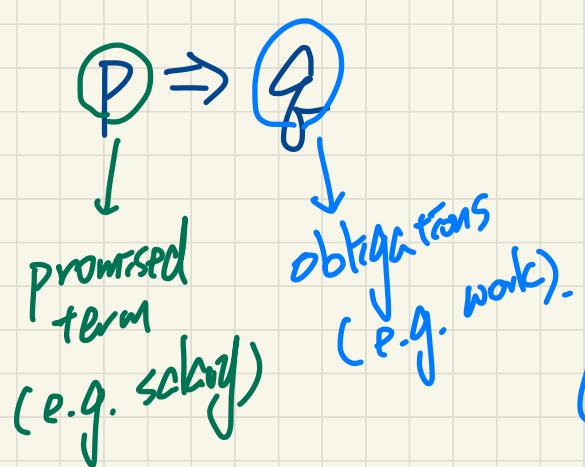
### Math Review

*Propositional Logic, Predicate Logic*

## Announcement

- Tuesday's lecture recording missing!
- Lab 1 released
  - + tutorial videos
  - + problems to solve

# Implication ≈ Whether a Contract is Honoured



$P \Rightarrow Q$  T if the contract  
is not breached

(C1)  $P = T \quad Q = T \quad \text{T}$

(C2)  $P = T \quad Q = F \quad \text{F}$

(C3)  $P = F \quad Q = T \quad \text{T}$

(C4)  $P = F \quad Q = F \quad \text{T}$

Common base / typical  
cases for justification.

# Expressing Implications

q if p, p is sufficient for q

casual replacement  
if p then q

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	false

conditions for  $\Rightarrow$  to be true  
C2:  $P \equiv F$

p: snow storm  
q: cancel class

q unless  $\neg p$

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	false

p only if q, q is necessary for p

what consequence is  
(the causal rel. not violated)

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	false

when cause is the not  $\top$   
then it doesn't matter

→ q is necessary to be  $\top$ , in  
order for  $\Rightarrow$  to be  $\top$   
also  $\top$  If we know p is  $\top$   
 $P \Rightarrow q \equiv \neg P \vee q$

Which of the following expressions  
are equivalent to  $P \Rightarrow q$

(1)  $q \leq f P$  ✓

(2)  $q \underline{\text{only if}} P$  X



$$P \Leftrightarrow q$$

$$\begin{aligned}\neg(P \wedge Q) &= \neg P \vee \neg Q \\ \neg(P \vee Q) &= \neg P \wedge \neg Q\end{aligned} \cdot P \Rightarrow Q$$

E.g.

$$x > 0 \wedge x \leq 23 \Rightarrow y \geq 23 \vee y < 46$$

Converse

$$y \geq 23 \vee y < 46 \Rightarrow x > 0 \wedge x \leq 23$$

Inverse:

$$\text{Equivalent proof in equational style}$$

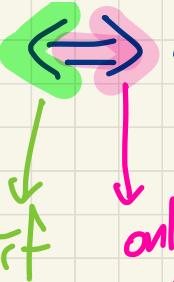
$$\neg(\neg P \Rightarrow \neg Q) \Rightarrow \neg(\neg P \wedge Q)$$

$\equiv \{\text{De Morgan's}$

$$\overline{| x \leq 0 \vee x > 23 \Rightarrow y < 23 \wedge y \geq 46|}$$

Prove

$$P \Leftrightarrow q$$



if      only if

Need to prove

(1)  $\underbrace{P \Leftarrow q}_{q \Rightarrow P} \quad P \underset{\text{if}}{\equiv} q$

(2)  $P \Rightarrow q \quad P \underset{\text{only if}}{\equiv} q$

## Identity

Identifiers

$$\boxed{0} + A = A$$

$$\boxed{1} * A = A$$

$$\text{True} \Rightarrow P \equiv P$$

as if:  
 $(\text{True} \Rightarrow P) \equiv P$

$$\text{True} \wedge P = P$$

$$\text{false} \vee P \equiv P$$

## Precedence of operators

$\neg$   
 $\wedge$   
 $\vee$   
 $\Rightarrow$   
 $\equiv$

## Zero

$$\text{false} \Rightarrow P \equiv \text{True}$$

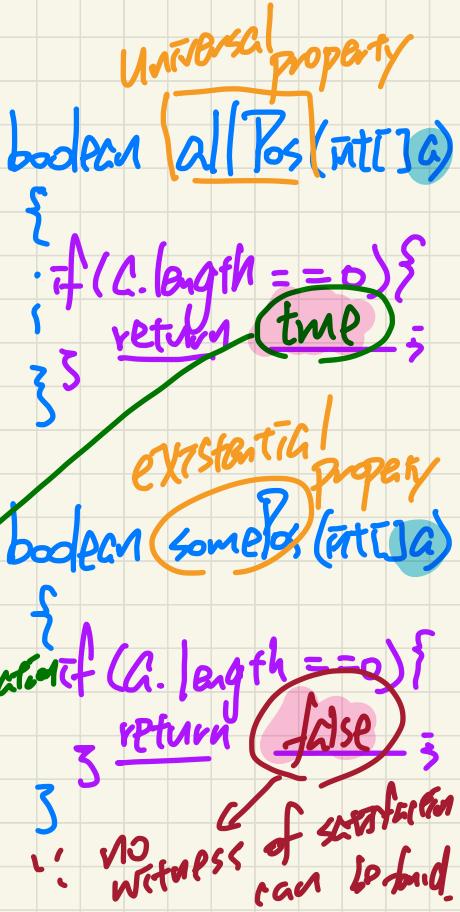
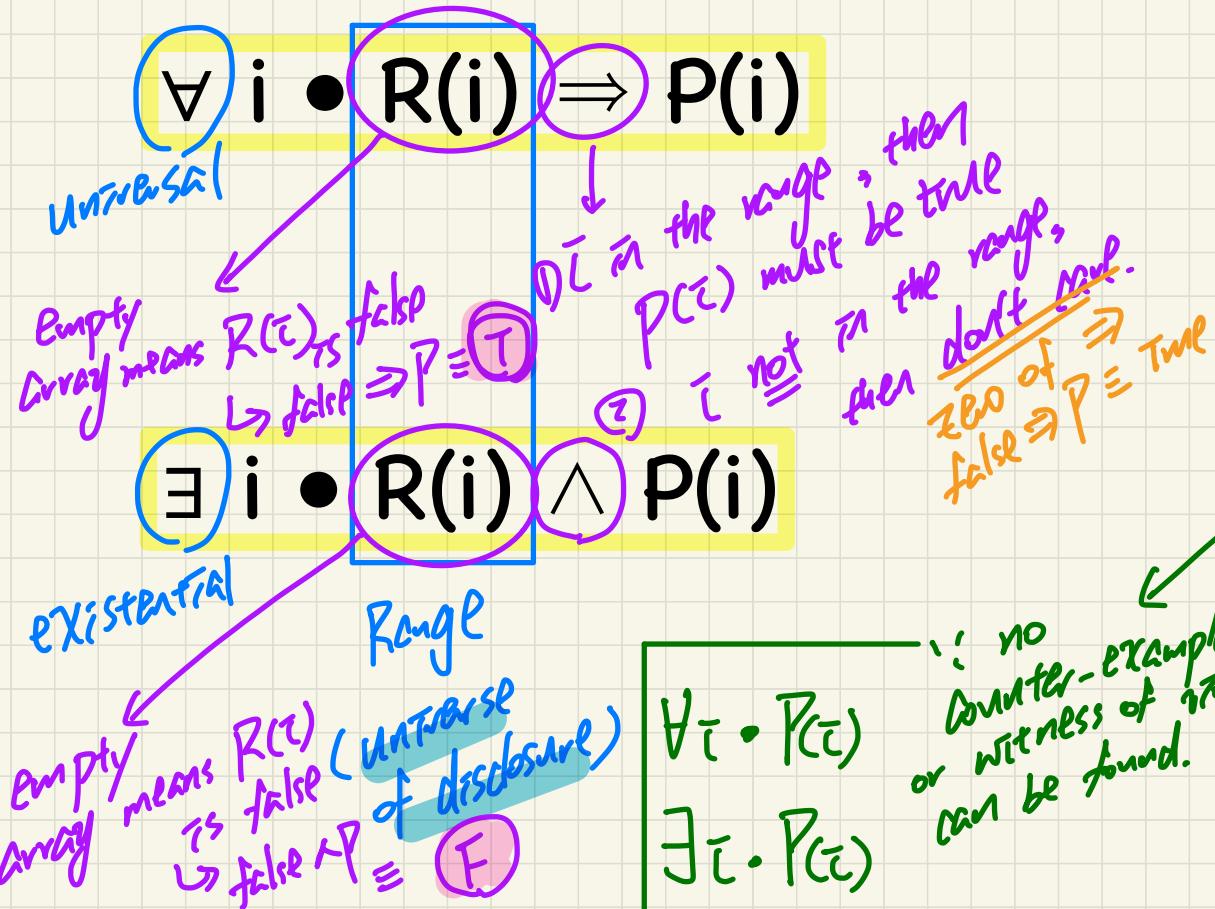
$$\text{false} \wedge P \equiv \text{false}$$

$$\text{true} \vee P \equiv \text{true}$$

# Predicate Logic: Quantifiers

- syntax

- base cases in programming



Родин

$$\exists^A \forall x : R(x) \Rightarrow P(x)$$

$$\exists \# \forall x : R(x) \wedge P(x)$$

✓

ИЛ

$$\forall A \forall x \in \text{Nat}, y \in \text{Int} \exists P(x) \ni P(x)$$

$$\forall E \forall x \in \text{Nat}, y \in \text{Int} \exists P(x) \ni P(x)$$

N

natural #s

↪ 0, 1, 2, 3, ... +∞

Z

integers

↪ -∞, ..., 0, ..., +∞

## Logical Quantifiers: Examples

✓  $\forall i \bullet i \in \mathbb{N} \Rightarrow i \geq 0$        $\stackrel{0, 1, 2, \dots}{=} \text{all elements in range} \geq 0$

✓  $\forall i \bullet i \in \mathbb{Z} \Rightarrow i \geq 0$       witness/counter-example:  $-2$   
 $-2 \in \mathbb{Z} \Rightarrow -2 \geq 0 \equiv T \Rightarrow F = \boxed{F}$

✓  $\forall i, j \bullet i \in \mathbb{Z} \wedge j \in \mathbb{Z} \Rightarrow i < j \vee i > j$

witness:  $\bar{i}=2, \bar{j}=2$

$\exists i \bullet i \in \mathbb{N} \wedge i \geq 0$        $\stackrel{\emptyset}{=} \text{witness: } 0$

✓  $\exists i \bullet i \in \mathbb{Z} \wedge i \geq 0$       witness:  $0$

$\exists i, j \bullet i \in \mathbb{Z} \wedge j \in \mathbb{Z} \wedge (i < j \vee i > j)$

$\hookrightarrow \bar{i}=1, \bar{j}=3$

$$\begin{array}{c} \frac{2 \in \mathbb{Z} \wedge 2 \in \mathbb{Z}}{T \wedge T \equiv T} \\ \Rightarrow \frac{2 < 2 \vee 2 > 2}{\boxed{F \vee F}} \\ \boxed{F} \end{array}$$